# Demonstration about the insolubility of the Solitaire Mancala

Luca Stornaiuolo – DOSE 2015, group 5

**Hypothesis:**

**3.1 Solitaire Mancala Rules**

***SR001: Initial Setup – Mancala board.*** The game shall be played using a standard Mancala board consisting of two rows, where each row has 6 holes and 1 store. The board is defined as:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | hole01 | hole02 | hole03 | hole04 | hole05 | hole06 | store02 |
| hole07 | hole08 | hole09 | hole10 | hole11 | hole12 |

***SR002: Initial Setup – initial configuration.*** The game shall be initiated with 48 stones randomly split into the 12 holes (hole01, hole02, … hole12; see SR001), leaving each hole with at least 1 stone (all 48 stones shall all be placed in the board, leaving the stores empty).

***SR003: Selection of hole before first round.*** The player shall select a hole (in any row, thus any of the 12 holes). SR004: A round. When a hole is selected, all stones in the selected hole are distributed either clockwise or anti-clockwise (the user can choose), as follows:

***SR004(A): Dropping clockwise.*** One stone is placed in each hole starting with the hole next to the selected one, in clockwise direction:

* If the number of stones remaining to be distributed is more than 1 after dropping in hole06 or hole07, then store02 or store01 respectively is skipped, and the next stone is dropped in hole12 or hole01 respectively. The round is over when there are no more stones to distribute. If the game is not over (see SR005 and SR006), a new round starts as 4 described in SR004, where the selected hole will be the one where the last stone of this round was dropped.
* If the number of stones remaining to be distributed is 1 after dropping in hole06, this stone is dropped in store02. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole06.
* If the number of stones remaining to be distributed is 1 after dropping in hole07, this stone is dropped in store01. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole07.

***SR004(B): Dropping anti-clockwise.*** One stone is placed in each hole starting with the hole next to the selected one, in anti-clockwise direction:

* If the number of stones remaining to be distributed is more than 1 after dropping in hole01 or hole12, the store01 or store02 respectively is skipped, and the next stone is dropped in hole07 or hole06 respectively. The round is over when there are no more stones to distribute. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be the one where the last stone of this round was dropped.
* If the number of stones remaining to be distributed is 1 after dropping in hole01, this stone is dropped in store01. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole01.
* If the number of stones remaining to be distributed is 1 after dropping in hole12, this stone is dropped in store02. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole12.

***SR005: Game over – player loses.*** When the last stone distributed in a round is placed in an empty hole, the player loses and the game is over.

***SR006: Game over – player wins.*** The player wins the game if no stone remains in any of the 12 holes.

**Thesis:**

**The player never wins.**

The condition described in paragraph SR006 is unreachable using game rules.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

**Definitions:**

**si**: valid game configuration (see SR001, SR002 and SR003);

**I**: set of initial configurations (see SR002);

**G**: set of game over losers configurations (see SR005);

**x**: number of stones | 0 ≤ x ≤ 48;

**S(si)**: set of valid successors of the game configuration si (a successor of si is a valid game configuration reachable from si using game rules (see SR004(A) and SR004(B)));

**P(si)**: set of valid predecessors of the game configuration si (a predecessor of si is a valid game configuration from which it is possible to reach si using game rules (see SR004(A) and SR004(B)));

**si → SR004(A): Dropping clockwise. → sj**: sj is reachable from si using game rules (see SR004(A));

**si → SR004(B): Dropping anti-clockwise. → sj**: sj is reachable from si using game rules (see SR004(B));

**hole**: is the selected hole.

e.g.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | **x** | x | x | x | x | store02 |
| x | x | x | x | x | x |

**Demonstration:**

**D1:**

- if selected hole is not equal at hole01, hole06, hole07, hole12 and the number of stones in the selected hole is equal to 1 then the configuration is a game over losers configuration.

s1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | **1** | x | x | x | x | store02 |
| x | x | x | x | x | x |

s2

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | **1** | x | x | x | store02 |
| x | x | x | x | x | x |

s3

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | x | **1** | x | x | store02 |
| x | x | x | x | x | x |

s4

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | x | x | **1** | x | store02 |
| x | x | x | x | x | x |

s5

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | x | x | x | x | store02 |
| x | **1** | x | x | x | x |

s6

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | x | x | x | x | store02 |
| x | x | **1** | x | x | x |

s7

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | x | x | x | x | store02 |
| x | x | x | **1** | x | x |

s8

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | x | x | x | x | x | x | store02 |
| x | x | x | x | **1** | x |

s1 , s2 , s3 , s4 , s5 , s6 ,s7 , s8  ϵ G(see SR004(A), SR004(B) and SR005)

**D2:**

- if si is a game over losers configuration then it has not successors.

S(s1), S(s2), S(s3), S(s4), S(s5), S(s6), S(s7), S(s8) ϵ Ø (see SR005)

**D3:**

- if si is a game over losers configuration then it is not a valid predecessor (see predecessor definition and D2).

si ϵ G => ∄ sj | si ϵ P(sj)

**D4:**

- to win the game you must reach the game over goal configuration s9 (see SR006).

s9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

**D5:**

- valid predecessors of the game over goal configuration s9 (see predecessor definition).

**P(s9):**

s10

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | **1** | 0 | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s10 → SR004(B): Dropping anti-clockwise. → s9

s11

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 0 | 0 | 0 | **1** | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s11 → SR004(A): Dropping clockwise. → s9

s12

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 0 | 0 | 0 | 0 | store02 |
| **1** | 0 | 0 | 0 | 0 | 0 |

s12 → SR004(A): Dropping clockwise. → s9

 s13

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | **1** |

s13 → SR004(B): Dropping anti-clockwise. → s9

**P(s10):**

s14

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | **2** | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s14 → SR004(B): Dropping anti-clockwise. → s10

s15

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | **1** | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s15 → SR004(B): Dropping anti-clockwise. → s10

s15 is not a valid predecessor of s10 (see D1 and D3)

**P(s14):**

s16

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | **1** | 1 | 0 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s16 → SR004(A): Dropping clockwise. → s14

s17

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 1 | **1** | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s17 → SR004(B): Dropping anti-clockwise. → s14

s17 is not a valid predecessor of s14 (see D1 and D3)

**P(s16):**

s18

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | **2** | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s18 → SR004(B): Dropping anti-clockwise. → s16

s19

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | **3** | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s19 → SR004(B): Dropping anti-clockwise. → s16

**P(s18):**

s20

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | **1** | 1 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s20 → SR004(A): Dropping clockwise. → s18

s20 is not a valid predecessor of s18 (see D1 and D3)

s21

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 1 | **1** | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s21 → SR004(B): Dropping anti-clockwise. → s18

s21 is not a valid predecessor of s18 (see D1 and D3)

**P(s19):**

s22

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | **1** | 2 | 0 | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s22 → SR004(A): Dropping clockwise. → s19

s22 is not a valid predecessor of s19 (see D1 and D3)

s23

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| store01 | 0 | 0 | 2 | **1** | 0 | 0 | store02 |
| 0 | 0 | 0 | 0 | 0 | 0 |

s23 → SR004(B): Dropping anti-clockwise. → s19

s23 is not a valid predecessor of s19 (see D1 and D3)

**D6:**

- s9 , s10 , s11 , s12 , s13 , s14 , s16 , s18 , s19  I

holei | stones of holei = 0 (see SR002)

**D7:**

- a state si is unreachable using game rules if

P(si) ϵ Ø *or* ( sj ϵ P(si) | sj  I *and* sj is unreachable using game rules)

(see SR002, SR003, SR004(A) and SR004(B))

**Thesis:**

- s9 is unreachable using game rules (see D5, D6 and D7)

(There are similar demonstrations for configurations s11, s12, s13.)

**Q.E.D.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 0 | 0 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | **1** | 0 | 0 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

… similar demonstrations …

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 0 | 1 | **1** | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | **1** | 1 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 0 | 2 | **1** | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | **1** | 2 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 0 | **2** | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 0 | **3** | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | **1** | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | **1** | 1 | 0 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | **2** | 0 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | **1** | 0 | 0 | 0 | 0 | x |
| 0 | 0 | 0 | 0 | 0 | 0 |